DETERMINING THE HEAT LOSSES DUE TO RADIATION IN AN AXIALLY SYMMETRIC HIGH-TEMPERATURE STREAM OF SPHERICAL PARTICLES

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UDC 536.242

The mean radiation losses in a diverging axially symmetric stream of spherical particles are determined on the basis of a probabilistic estimate.

The study of high-temperature heat transfer involving particles or droplets is of great interest in many areas of science and engineering. This problem is especially important in processes such as the casting of metals and the combustion of powder fuel.

In practical calculations of radiative heat transfer in dispersed media bulk parameters of radiation are used: the dispersivity and the dispersion indicatrix. Such calculations applicable to various heat-engineering devices have been performed in [1]. For several dispersion systems, however, simple estimates of radiation losses can be made on the basis of probabilistic concepts associated with the shielding of particles during the heat transfer.

For our analysis we choose the most important practical case, that of a steady axially symmetric diverging stream of freely falling spherical particles or droplets. It will be assumed that all particles or droplets are of the same size and that heat losses can be incurred only by radiation into the unshielded surrounding space. For simplicity, we will henceforth refer to particles only.

We define a random parameter of this dispersion system. For this purpose we single out a cylindrical layer of height $2r_0$ within the stream of particles and track its downward motion during an arbitrary interval of time. The heat loss from this layer is made up of the heat losses from its component particles. The contributions of different particles will not be the same, however. Thus, an arbitrary particle located at the periphery of the stream will contribute most to the heat losses from the layer while a particle at the center of the stream will contribute least. This picture is explained by the shielding of the radiating surface of particles by neighboring particles and it determines the existence of a discretely distributed random parameter in the form of a relative shielded surface area of a particle.

Using the axial symmetry of the system, we will divide all particles within the selected stream layer into groups on the basis of this parameter, combining particles with equal shielded surface areas, and we will refer all properties of a particle which depend on its position in the stream to the coordinate of its center of gravity.

Let r_j be the center-of-gravity coordinate and $P_j(r_j, R)$ be the shielding probability of particles in the j-th group. The function $P_j(r_j, R)$ will be defined as the probability that a particle with its center of gravity at point r_j within the selected stream layer of radius R has a shielded surface $4\pi r_0^2 P_j(r_j, R)$. Given the number of particles n_j in every group, we can find the mean probability of surface shielding for a particle in any arbitrary stream layer and, after averaging the result over all stream layers, we can determine the mean probability of surface shielding for a particle during its flow time:

$$P_{sh} \equiv \frac{\langle x \rangle}{4\pi r_0^2} = \left\langle \frac{\sum_j n_j P_j(r_j, R)}{\sum_j n_j} \right\rangle, \quad j = 1, 2, \ldots, l = \frac{R_0}{r_0} - 1. \tag{1}$$

N.I. Lobachevskii Gor'kii State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 5, pp. 815-820, November 1971. Original article submitted December 29, 1970.

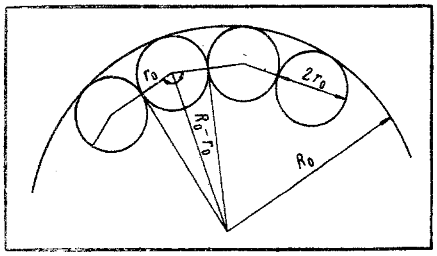

Fig. 1. Distribution of peripheral particles in a group in a stream layer.

The flow time is fixed by the finite radius R(t) of the diverging stream of particles. In order to establish the function $P_j(r_j, R)$, we use a geometrical model of the packing of spherical particles in an elementary cylindrical tube of height equal to the particle diameter. We now introduce the mean packing density of particles in the form of the rarefaction parameter of a layer $R/R_0 = \beta$, which is defined continuously within the region $1 \le \beta < \infty$. Let us examine a layer with a dense packing of particles, where $R = R_0$. In this case the function $P_j(r_j, R)$ is everywhere equal to unity, except at the peripheral particles. Let us find the boundary value $P_l(r_l, R_0)$ on the basis of the geometrical model shown in Fig. 1:

$$P_l(r_l, R_0) \equiv P_0 = \frac{2\arccos\dfrac{r_0}{R_0 - r_0}}{2\pi}.$$

This means that $P_j(r_j, R_0)$ is a step function defined in a discrete parameter space and assuming its characteristic value at the layer boundary ($r_j = r_l = R_0 - r_0$):

$$P_j(r_j, R_0) = \begin{vmatrix} 1, & j \ne \dfrac{R_0}{r_0} - 1, \\ \dfrac{1}{\pi}\arccos\dfrac{r_0}{R_0 - r_0}, & j = \dfrac{R_0}{r_0} - 1. \end{vmatrix} \tag{2}$$

We next convert to dimensionless quantities:

$$\beta_j = \frac{r_j}{R_0}, \qquad \nu_0 = \frac{R_0}{r_0}$$

and determine the form of function $P_j(\beta_j, \beta_0 = 1)$. It follows from condition (2) for $j \ne \nu_0 - 1$ that

$$\frac{\partial P_j(\beta_j, \beta_0)}{\partial \beta_j} = 0,$$

$$1 - P_j(\beta_j, \beta_0) = 0.$$

Combining these two relations through indeterminate coefficients η, γ, and then integrating the resultant equation, we have

$$P_j(\beta_j, \beta_0) = 1 - C\exp - \gamma(\beta_j + \eta). \tag{3}$$

To this solution we add the boundary condition

$$P_l(\beta_0 - \nu_0^{-1}, \beta_0) = P_0$$

and obtain

$$P_0 = 1 - C, \qquad \beta_0 - \nu_0^{-1} + \eta = 0. \tag{4}$$

Furthermore, since $P_j(\beta_j, \beta_0) = 1$ for $j \ne \nu_0 - 1$ ($\beta_0 \ge \beta_j + \nu_0^{-1}$), hence $\gamma(\beta_0 - \beta_j - \nu_0^{-1}) = \infty$ and

$$\gamma = \frac{1}{\beta_0 - 1}. \tag{5}$$

With the aid of expressions (3), (4), and (5), we can transform condition (2) to

$$P_j(\beta_j, \beta_0) = 1 - (1 - P_0)\exp - \frac{\beta_0 - \beta_j - \nu_0^{-1}}{\beta_0 - 1}.$$

For all states of the stream layer, with $\beta > \beta_0$, the step function $P_j(\beta_j, \beta_0)$ is characteristically smoothed out by

$$P_j(\beta_j,\ \beta) = 1-(1-P_0)\exp-\frac{\beta-\beta_j-\nu_0^{-1}}{\beta-1}. \tag{6}$$

Assuming that the particles are small ($\nu_0 \gg 1$), we determine the mean probability $P_j(\beta_j,\ \beta)$; we have

$$P_{sh}(\beta) \equiv \langle P_j(\beta_j,\ \beta)\rangle = 1-(1-P_0)E(\beta), \tag{7}$$

where

$$E(\beta) \equiv \langle \exp-\frac{\beta-\beta_j-\nu_0^{-1}}{\beta-1}\rangle = 2\left(\frac{\beta-1}{\beta-\nu_0^{-1}}\right)\left\{\frac{1-\nu_0^{-1}}{\beta-1}+\exp-\frac{\beta-\nu_0^{-2}}{\beta-1}\right\}. \tag{8}$$

Since the rarefaction parameter for a stream layer is defined continuously within the region $1 \le \beta < \infty$, the probability $P_{sh}(\beta)$ may be averaged over the interval $1-\beta_f$. Namely,

$$P_{sh} \equiv \langle P_{sh}(\beta)\rangle = 1-(1-P_0)S(\beta_{sh}), \tag{9}$$

where

$$S(\beta_f) \equiv \langle E(\beta)\rangle = 2\left\{\frac{1-\nu_0^{-1}}{\beta_f-1}\ln\frac{\beta_f-\nu_0^{-1}}{1-\nu_0^{-1}}-\frac{1-\nu_0^{-1}}{\beta_f-\nu_0^{-1}}+\frac{1}{\beta_f-1}\int_1^{\beta_f}\left(\frac{\beta-1}{\beta-\nu_0^{-1}}\right)^2\exp\left(-\frac{\beta-\nu_0^{-1}}{\beta-1}\right)d\beta\right\}. \tag{10}$$

The integral in (10) can be easily expressed in terms of an integral exponential function:

$$\frac{1}{\beta_f-1}\int_1^{\beta_f}\left(\frac{\beta-1}{\beta-\nu_0^{-1}}\right)^2\exp\left(-\frac{\beta-\nu_0^{-1}}{\beta-1}\right)d\beta = \frac{\beta_f+1-2\nu_0^{-1}}{\beta_f-\nu_0^{-1}}\exp\left(-\frac{\beta_f-\nu_0^{-1}}{\beta_f-1}\right)$$

$$-\frac{1-\nu_0^{-1}}{\beta_f-1}\,\mathrm{Ei}\left(-\frac{\beta_f-\nu_0^{-1}}{\beta_f-1}\right)+\frac{3(1-\nu_0^{-1})}{e(\beta_f-1)}\,\mathrm{Ei}\left(-\frac{1-\nu_0^{-1}}{\beta_f-1}\right). \tag{11}$$

Inserting (10) and (11) into (9) yields

$$P_{sh} = 1-(1-P_0)\,2\left\{\frac{1-\nu_0^{-1}}{\beta_f-1}\ln\frac{\beta_f-\nu_0^{-1}}{1-\nu_0^{-1}}-\frac{1-\nu_0^{-1}}{\beta_f-\nu_0^{-1}}\right.$$

$$\left.+\frac{\beta_f+1-2\nu_0^{-1}}{\beta_f-\nu_0^{-1}}\exp\left(-\frac{\beta_f-\nu_0^{-1}}{\beta_f-1}\right)-\frac{1-\nu_0^{-1}}{\beta_f-1}\,\mathrm{Ei}\left(-\frac{\beta_f-\nu_0^{-1}}{\beta_f-1}\right)+\frac{3(1-\nu_0^{-1})}{e(\beta_f-1)}\,\mathrm{Ei}\left(-\frac{1-\nu_0^{-1}}{\beta_f-1}\right)\right\}. \tag{12}$$

According to this expression, the mean probability of a particle surface being shielded during its flow time P_{sh} is determined by the relative particle dimension ν_0^{-1} and the final value of the rarefaction parameter β_f for a stream layer. Equation (12) is represented graphically in Fig. 2. Using the initial expression (1), we now find the mean value of the surface shielding of a particle:

$$\langle x\rangle = 4\pi r_0^2 P_{sh}. \tag{13}$$

A particle having such a shielded surface area will be called a characteristic particle; its specific heat losses correspond to the specific heat losses of the entire stream.

Let us then determine the losses from an unshielded particle, using the solution given in [2] to the problem of heat conduction in a sphere with uniform Stefan–Boltzmann boundary conditions and considering that $\overline{T}(r,\ t-\tau) \gg \overline{T}_a$,

$$\frac{\overline{T}(r,\ t-\tau)}{\overline{T}(\tau)} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2\mathrm{Bi}\{\mu_n^2+(\mathrm{Bi}-1)^2\}^{\frac{1}{2}}r_0\sin\mu_n\frac{r}{r_0}}{\mu_n(\mu_n^2+\mathrm{Bi}^2-\mathrm{Bi})r}\exp(-\mu_n^2\,\mathrm{Fo}). \tag{14}$$

Averaging the temperature over the volume and taking into account that $\mathrm{Bi} \ll 1$, we find

$$\langle \overline{T}(t-\tau)\rangle = \overline{T}(\tau)\exp(-3\,\mathrm{Bi}\,\mathrm{Fo}). \tag{15}$$

The Bi number in this expression does not remain constant during the flow time of particle, because the temperature of its radiating surface changes. Therefore, Bi may be replaced by its average value $\langle \mathrm{Bi}\rangle$ over the time interval $T-\tau$, and this value is found by solving (14) for $r = r_0$ and $\mathrm{Bi} \ll 1$:

$$\left(\frac{\langle \mathrm{Bi}\rangle}{\mathrm{Bi}}\right)^{\frac{1}{3}} = \frac{\sin\sqrt{3\langle \mathrm{Bi}\rangle}}{\sqrt{3\langle \mathrm{Bi}\rangle}}\left(\frac{1-\exp(-3\langle \mathrm{Bi}\rangle\mathrm{Fo})}{3\langle \mathrm{Bi}\rangle\mathrm{Fo}}\right). \tag{16}$$

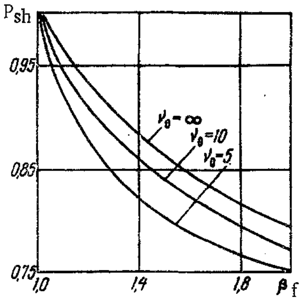

Fig. 2. Probability P_{sh} as a function of the rarefaction parameter β_f for a comminuted melt jet.

According to [3], the initial value of the Bi number is

$$\mathrm{Bi} = c_0\varepsilon\left(\frac{T(\tau)}{100}\right)^3\frac{r_0}{\lambda}. \tag{17}$$

In terms of (15), consequently, the specific heat losses from an unshielded particle are

$$Q = c\overline{T}(\tau)\{1 - \exp(-3\langle \mathrm{Bi}\rangle \mathrm{Fo})\}. \tag{18}$$

Furthermore, $\langle \mathrm{Bi}\rangle = \langle \alpha\rangle r_0/\lambda$ with $\langle\alpha\rangle = c_0\varepsilon(\langle T\rangle/100)^3$. In the case of a shielded particle the concept of the equivalent Bi number may be introduced:

$$\langle \mathrm{Bi}\rangle_{sh} = \frac{\langle\alpha\rangle_{sh} r_0}{\lambda},$$

where $\langle\alpha\rangle_{sh} = (1 - P_{sh})\langle\alpha\rangle$, i.e.,

$$\langle \mathrm{Bi}\rangle_{sh} = (1 - P_{sh})\langle \mathrm{Bi}\rangle. \tag{19}$$

In this way, with the aid of (18) and (19), the heat losses from a shielded particle can be expressed as

$$Q_{sh} = c\overline{T}(\tau)\{1 - \exp[-3(1 - P_{sh})\langle \mathrm{Bi}\rangle \mathrm{Fo}]\}. \tag{20}$$

For a high-temperature system of droplets where the latter may be cooled below their freezing point, it becomes necessary, in view of the stipulations on the solution of (14), to impose on (20) the restriction that the state of the medium cannot change. This requires that

$$\langle \mathrm{Bi}\rangle > \langle \mathrm{Bi}_L\rangle, \tag{21}$$

where the value of the limiting number $\langle \mathrm{Bi}\rangle_L$ is determined from the expression

$$\left(\frac{\langle \mathrm{Bi}\rangle_L}{\mathrm{Bi}}\right)^{\frac{1}{3}} = \frac{\dfrac{\sin\sqrt{3\langle \mathrm{Bi}\rangle_L}}{\sqrt{3\langle \mathrm{Bi}\rangle_L}} - \dfrac{\overline{T}_s}{\overline{T}(\tau)}}{\ln\left(\dfrac{\sin\sqrt{3\langle \mathrm{Bi}\rangle_L}}{3\langle \mathrm{Bi}\rangle_L}\cdot\dfrac{\overline{T}(\tau)}{\overline{T}_s}\right)}, \tag{22}$$

which can easily be derived by averaging solution (14) for $r = r_0$ and $\mathrm{Bi} \ll 1$.

As an example, let us determine the heat losses from a jet of comminuted molten stainless steel with the following values for the parameters needed in the calculation: $R_0 = 15 \cdot 10^{-3}$ m; $\nu_0 = 10$, $T(\tau) = 2073$°K; $\beta_f = 1.1$; height of the comminuted melt jet 0.3 m.

According to [4], the thermophysical characteristics of interest here are: $\lambda = 29.1$ W/m · deg; $c = 0.84$ KJ/kg · deg C; $a = 0.54 \cdot 10^{-5}$ m²/sec; $T_s = 1743$°K; $c_0\varepsilon = 2.60$ W/m² · deg⁴.

At a pouring rate of 1 m/sec for metals, the flow time of a characteristic droplet is 0.3 sec, i.e., Fo = 1.62. With the aid of (14) and (17), we find $P_{sh} = 0.95$ and $\mathrm{Bi} = 3.22 \cdot 10^{-2}$. A graphical solution of Eq. (16) yields $\langle \mathrm{Bi}\rangle = 2.61 \cdot 10^{-2}$. According to Eq. (22), the $\langle \mathrm{Bi}\rangle_L$ number is equal to $2.49 \cdot 10^{-2}$ and, consequently, requirement (21) is satisfied. Equation (20) yields the heat losses from a comminuted melt jet: $Q_{sh} = 9.3$ KJ/kg

If one assumes $\beta_f = 2$ here, with all other conditions as before, then, $P_{sh} = 0.775$ and $Q_{sh} = 42.3$ KJ/kg.

NOTATION

r_j is the coordinate of the center of gravity of a particle in the stream layer;
n_j is the number of particles in the group with coordinate r_j;

n	is the total number of particles in the stream layer;
$P_j(\beta_j, \beta)$	is the probability of surface shielding for an arbitrary particle;
$P_{sh}(\beta)$	is the mean shielding probability for a particle in the stream;
P_0	is the shielding probability for a particle at the stream boundary;
R_0	is the radius of the stream layer at the initial moment;
R	is the radius of the stream layer at any instant of time t;
$t_f - \tau$	is the flow time of a particle;
r_0	is the radius of a particle;
$T(r, t-\tau)$	is the instantaneous temperature at an arbitrary point of the particle;
$\overline{T}(r, t-\tau) = T(r, t-\tau) - 273°K$	
T_a	is the ambient temperature;
$\langle T(t-\tau)\rangle$	is the average temperature of a particle;
$T(\tau)$	is the initial temperature of particles in the stream layer;
T_s	is the temperature of solidification;
c	is the specific heat;
ρ	is the density;
a	is the thermal diffusivity;
c_0	is the radiation constant;
ε	is the emissivity;
λ	is the thermal conductivity;
Q_{sh}	is the specific heat loss from a shielded particle;
Fo	is the Fourier number;
Bi	is the Biot number;
Q	is the specific heat loss from an unshielded particle;
$\langle x\rangle$	is the mean shielded surface area of a particle;
$\langle Bi\rangle_e$	is the equivalent Biot number;
$\langle Bi\rangle_L$	is the limiting Biot number;
P_{sh}	is the mean probability of surface shielding for a characteristic particle.

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